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Ranging With Electromagnetic Singularities

by Joseph N. Mait, Canh Ly, and Markus Testorf

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14. ABSTRACT

Most improvements in radar have concentrated solely on the temporal characteristics of the electromagnetic pulse. The spatial dimension is typically exploited only through arrays of antennas. The limits to using spatial characteristics of a transmitted beam to perform ranging were investigated, specifically, Laguerre-Gaussian beams as potential carriers for encoding range in spatial characteristics. The results indicate that whereas the beams are indeed capable of encoding range, aberrations and imaging fidelity limit the utility of the approach to small ranges. The system is better suited for microscopy than it is for battlefield ranging.

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ranging, Laguerre-Gaussian beams, optical singularities, Laguerre-Gaussian modes, encoding

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1. Objective

The proposed work seeks to reduce the complexity of radar ranging systems without a concomitant reduction in resolution by exploiting spatial beam characteristics as opposed to temporal. A key objective is demonstrating ranging capabilities using electromagnetic waveforms that have singularities. If the principle is validated, the hardware and processing costs required to achieve a particular range resolution will be addressed.

2. Approach

Low-cost solutions to traditional problems in imaging exist if the optics and the postdetection processing are designed jointly. This philosophy, joint design between sensor front ends and signal processing, drives this proposal. By taking advantage of the spatial characteristics of a radar or ladar beam profile, it may be possible to reduce the complexity of the temporal processing or enhance the performance of a low-cost system. This is achieved by employing electromagnetic beams that contain phase vortices or dislocations (*1–3*).

Such singularities can be imposed upon a beam by modifying its phase using a dielectric element or phasing the elements of an antenna array. For example, a phase function that varies linearly with angle, $\phi(r,\varphi,z) = \exp(il\varphi)$, ($l$ an integer) imposes an amplitude null on an incoming beam. Since the spatial phase is a function of angular position to the optic axis, as the beam propagates, its lines of constant phase create a helix. A beam with $l = 1$ is a vortex of order one.

As a model system, Laguerre-Gaussian (LG) modes as specific examples of beams that contain a screw wavefront dislocation (*1*, *2*) are considered as follows:

$$f_{l,m}(\rho,\varphi,z) = G(\rho,\phi,z)\left(\frac{\sqrt{2}\rho}{w(z)}\right)^{m} L_l^{|m|}\left(\frac{2\rho^2}{w^2(z)}\right)\exp\left[j\phi(\rho,\varphi,z)\right], \tag{1}$$

where $G(\rho,\phi,z)$ is a Gaussian beam and $L_m^n(x)$ are generalized Laguerre polynomials of order (m,n). The phase of the beam,

$$\phi(\rho,\varphi,z) = \exp\left(ikz + i\frac{k\rho^2}{2R(z)} + il\varphi - iQ\arctan\left(z/z_R\right)\right), \tag{2}$$

describes a helical wavefront with a pitch of $|l|\lambda$.

Range information compatible with standard radar systems from the last term of the phase function can be obtained, i.e., the Gouy phase shift. For different *Q*, the rotation angle of the wavefront is out of phase after propagating a certain distance. Range information is reflected in the phase difference for different *Q*.

Piestun et al. (*3*) have shown that if a designer constructs a pattern $u(\rho,\varphi,z)$ as a superposition of singular beams with different *Q*,

$$u(\rho,\varphi,z)=\sum_{k=1}^{K}a_k f_{l_k m_k}(\rho,\varphi,z), \tag{3}$$

its intensity distribution $\left|u(\rho,\varphi,z)\right|^2$ rotates slowly about the optical axis as it propagates through the beam waist. That is, rotation of the intensity distribution translates directly into range.

It is believed that the design of the intensity of the beam cross section will minimize the cost of postdetection processing to obtain range information as well as the impact of aberrations caused by air turbulence and surface properties of the target on the system performance.

3. Results

First-year modeling validated that if a designer constructs an intensity pattern as a superposition of singular beams with different values of ℓ and *Q*, the intensity distribution rotates slowly about the optical axis as it propagates through the beam waist. That is, rotation of the intensity distribution translates directly into range.

The modeling and simulation of the ranging system from the first year were complemented in the second year with experiments aimed at investigating systems properties. As opposed to using RF frequencies, the experiments were performed at optical frequencies, thus allowing the construction of a compact system. Further, the accessibility of sources and detectors at optical frequency facilitated the assessment of system properties with respect to source and detector properties. The availability of dynamic devices also provided considerable flexibility in generating and testing different spatial patterns. (Such spatial modulators are not currently available at RF frequencies.) As a result, it was possible not only to investigate the feasibility of the basic principle but to experimentally investigate the limits of performance.

Experimental implementation requires a Gaussian beam envelope modulated in magnitude and phase to implement the superposition of Laguerre-Gaussian modes. A Gaussian envelope is readily available as the output mode of a conventional laser. The high-order Laguerre-Gaussian modes can be generated by placing a diffractive optical element in the beam path. Instead of a fixed element, a Holoeye LC-2500 reflective spatial light modulator (SLM) (*4*) was used to provide flexibility in implementing different modulation patterns. The device has 768 × 1024 pixels, each with 8 bits of phase modulation. Phase-only operation insures maximum energy throughput but requires one to encode the superposition of LG modes $u(\rho,\varphi,z)$, which is in general complex. Issues related to the beam synthesis and encoding were reported at two conferences (*5*, *6*).

To encode magnitude and phase with a phase-only element requires at least two pixels of the SLM, one each for each degree of freedom of the desired distribution. The coding scheme is illustrated in figure 1a. The complex amplitude u is represented as the vector sum of two unit phasors with phase ϕ_1 and ϕ_2.

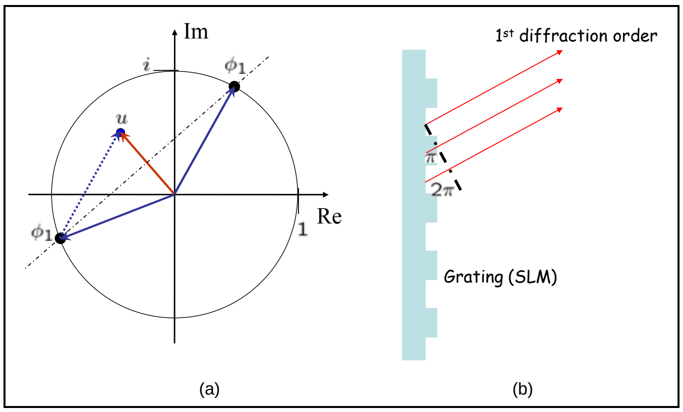

Figure 1. Phase-only encoding to realize an arbitrary complex value: (a) graphical representation in phasor-space and (b) implementation as an encoded grating on the SLM.

To observe the desired pattern $u(\rho,\varphi,z)$ optically, the complex amplitude of the LG beam superposition with a linear grating needs to be modulated. As illustrated in figure 1b, the phase difference of the wavefront emerging from the grating is 2π per transverse period. Consequently, in order to collect maximum energy in the first diffraction order, a phase shift of π per each half period needs to be ensured. This can be achieved with a binary grating phase grating with duty cycle 1:1 and π-phase modulation. In other words, π-phase must be added to one of the two phase values, ϕ_1 and ϕ_2.

The phase-only distribution written onto the SLM does not encode the Gaussian beam envelope. The expression for LG beams indicates that the Gaussian beam is decoupled from the rest of the complex amplitude distribution. This implies that the incident beam should be Gaussian in profile. In this case, the location of the beam waist can be chosen using a beam expander without changing the phase modulation.

The optical system is represented in figure 2. The laser beam, which outputs 15 mW at $\lambda = 633$ nm wavelength, is spatially filtered by a 10-µm-diameter pinhole and expanded using an inverse Kepler telescope. The Gaussian beam diverging from the pinhole is transformed by a 50-mm focal-length lens into a converging beam whose waist is located approximately 2 m from the lens.

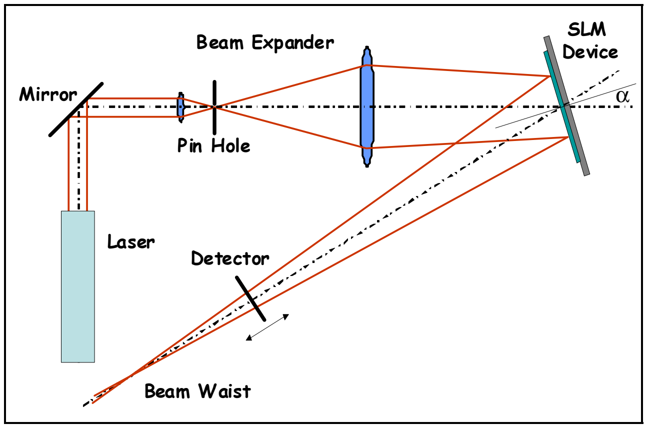

Figure 2. Schematic of the optical experiment used to test LG modes for ranging.

The output polarization of the laser was adjusted to vertical linear to ensure that the SLM operated in its phase-only modulation mode. The SLM is located in the converging beam at a distance that allows the incident and reflection angle α to be small. This guarantees that unspecified polarization effects at the SLM surface have a minimal effect on the beam that would otherwise be evident at large incident angles.

To ensure proper implementation of the desired phase function, the SLM device has to be calibrated with respect to the phase modulation that can be achieved at a particular wavelength. A binary-phase grating with duty cycle 1:1 to estimate the modulation depth was used. Such a grating should generate no energy on axis. In contrast, the zeroth diffraction order was never completely suppressed regardless of the modulation, which was attributed qualitatively to a pixel fill factor less than one. Minimum intensity of the zeroth order was achieved for modulation values of about 170 to 180 (on a scale from 0 to 255). This means the maximum phase modulation that could be programmed was $0.7 \times 2\pi = 1.4\pi$. This value was confirmed using an interferometric analysis of the SLM.

This phase limitation needs to be accounted for in the encoding scheme. Recall that the encoding requires adding a π-phase shift to one of the two phase values. Thus, adding the phase to the value ensures that both phases are less than 1.4π and allows most of the values contained in the unit circle in the complex plane to be covered.

The three-spot pattern represented in figure 3 was selected for the experiments. The encoding scheme described in the previous section resulted in the phase-only distribution represented in figure 4.

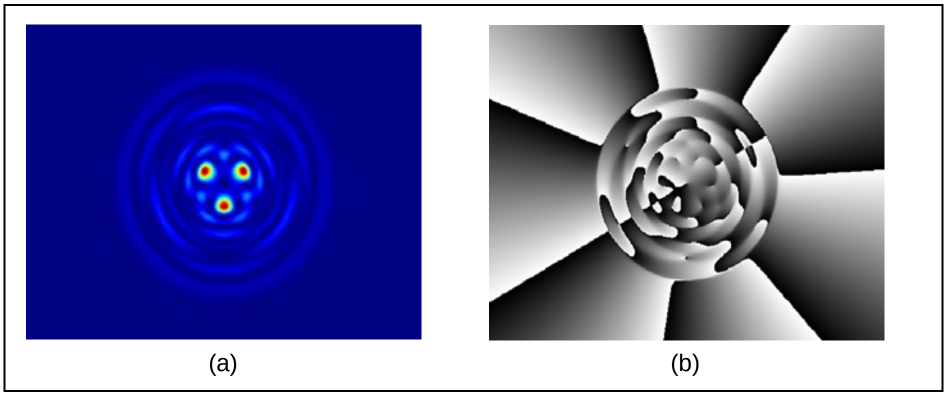

Figure 3. Three-spot pattern used in experiments: (a) magnitude and (b) phase of the pattern.

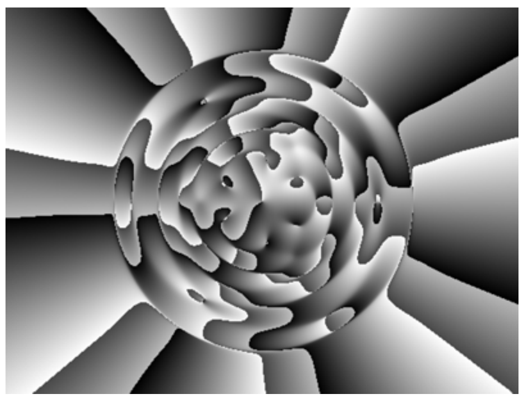

Figure 4. Gray-scale representation of the phase distribution written onto the SLM. The phase ranges from 0 (black) to 1.4π (white).

The basic properties of the rotating beam pattern were verified by recording images of the beam cross section over 2.4 m at 10-cm increments. A subset of the recorded images is shown in figure 5. The results confirm the rotation of the three-spot pattern as a function of distance. This includes the scaling of the beam cross section in accordance with the diameter of the converging (diverging) beam in front of (behind) the waist.

The recorded images also reveal problems close to the beam waist. The spot pattern is hardly visible due to strong artifacts. This is interpreted as a consequence of aberrations in the beam envelope due to an imperfect Gaussian shape. At the beam waist, the pattern can be interpreted as the ideal pattern convolved with the Fourier transform of the aberration in the plane of the SLM. The system appears to be highly sensitive to aberrations. Further study is required for a full quantitative understanding of this behavior.

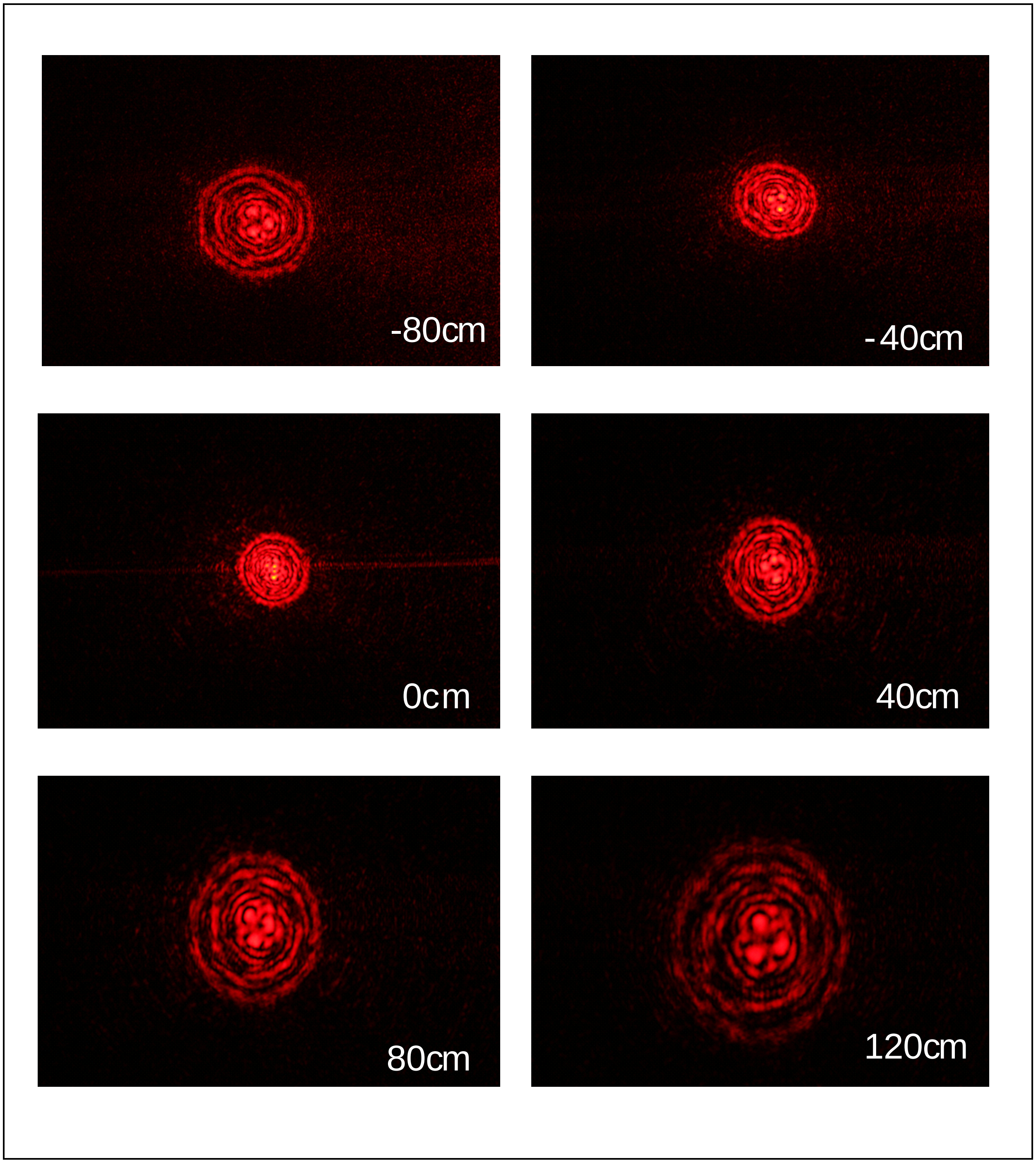

Figure 5. Experimental results of the rotating beam pattern. The labels indicate the distance from the plane of the waist.

A second series of experiments was conducted to investigate the impact of atmospheric turbulence. To this end, an electric air heater was located underneath the beam to generate a flow of hot and turbulent air. A comparison between images with the heater on and off indicated that rapid fluctuations exist in the pattern. Qualitative observation of the fluctuations revealed a clear shift of one or more spots. The latter was often manifested as a small rotation in the entire pattern. Such observations indicated potential difficulties in determining the rotation angle and thus obtaining the range from these images.

Although these experiments exhibited a clear impact of a turbulent atmosphere on the beam propagation, these effects were small in comparison to those due to beam aberrations. Aberration effects close to the plane of the waist were of concern since the waist would be used as the working point for a ranging system.

4. Conclusions

An unconventional means for determining range resolution was proposed by converting the Gouy phase of a superposition of LG beams into a rotating intensity pattern. The patterns to generate LG beams on a phase-only SLM at optical frequencies were implemented. Issues related to pattern synthesis and encoding to ensure proper pattern generation were investigated.

The experiments indicated that pattern generation was sensitive to aberrations, which increased the complexity of the optical system to produce the patterns and increased the complexity of the signal processing required to derive range from angular rotation.

Thus, the system was not robust enough for the original application as a replacement for radar. Nonetheless, it has merit but might better be suited to applications in microscopy.

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